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POSSIBILITIES FOR CONSTRUCTING A UNIFIED FAILURE THEORY

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The abundance of existing and newly developed materials and the various conditions for using them has led to creation of numerous, as a rule, semiempirical theories, criteria, concepts of failure, each of which holds for an experimentally studied range of change in parameters. These special theories together with previous experience of strength analysis have made it possible for a certain time to be limited to them. However, further development of technology in the direction of creating large unique objects intended for operating under conditions of intense dynamic loads, the impossibility in a number of cases of carrying out full-scale tests for these objects in order to explain their actual strength margins, and also continuing cases of unpredicted catastrophic failure for certain objects built in accordance with existing strength standards, require not so much development and creation of new failure criteria, as the requirement of finding a single physically substantiated approach to the problem as a whole, if only at the level of phenomenology without considering the fine details of failure phenomena and complicating circumstances. This theory with a capacity to some extent or other to combine special criteria (concepts) for failure should be built up taking account of the generally accepted fact, i.e., failure calculated for the whole in parts, is completion of work in proportion to the fracture surface. Therefore, work and energy specific for a unit of surface should act as criterial values. In fact, use of an energy approach with local consideration of conditions of a changeover of a crack to unsteady growth explains the vigorous development and success in understanding many details and features of brittle failure achieved by fracture mechanics (FM). Attempts to use FM for describing other forms of failure have been fruitful. However, it is not in a state of combining and describing all forms of it [1, 2].

Recently in works by the author with co-workers, and also by other domestic and overseas researchers a study has been carried out of failure for dynamically loaded shells. On the one hand these studies have made it possible to reveal a number of new effects not found in FM, and on the other hand, based on energy balance applied to the whole object in question or specified parts of it, to describe these phenomena and understand their physical nature. In future we call this the integral approach (IA) in contrast to the local approach used in FM. The integral approach makes it possible to look at the problem as a whole and to find a scheme for constructing an overall theory for failure. Previously such an attempt using the IA was made in [3]. Studies performed subsequently using the IA [3-14] provide a basis for its fruitfulness and necessity of developing it further.

We consider failure of a material cube with edge L stretched by forces σL^2 at two opposite faces. The rest of the faces are free. We also assume a piecewise linear rule for material deformation and it consists of an elastic region where

$$\sigma = \varepsilon E \tag{1}$$

up to $\sigma = \sigma_y$, where σ_y is yield stress (and elastic limit) for the material and region for plastic strain ($\sigma > \sigma_y$):

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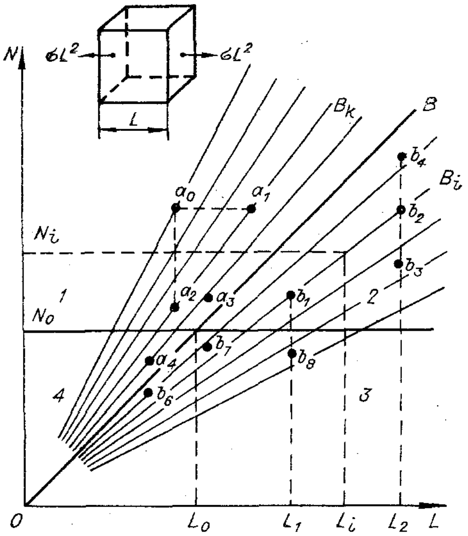

Fig. 1

$$\sigma = \sigma_y + K(\varepsilon - \sigma_y/E) \quad (2)$$

(E and K are Young's and strengthening moduli).

We turn attention to brittle failure occurring in small time intervals comparable with $t \sim L/c$ (where c is sound velocity in the material). Work of external forces σL^2 for the time of failure may be ignored and it may be assumed that in completing the work of separation λL^2 only the reserve of elastic energy (EE) may be expended existing at the start of failure in the cube $\sigma^2 L^3/2E$ (λ is specific work for failure per unit of surface). It is evident that a required condition for failure, or nonfailure, is written as

$$\sigma^2 L^3/(2E) \geqslant \lambda L^2; \quad (3)$$

$$\sigma^2 L^3/(2E) < \lambda L^2. \quad (4)$$

Expression (3) on coordinates N – L ($N = 2\lambda E/\sigma^2$) in Fig. 1 corresponds to the region LOB also including the line OB, and inequality (4) corresponds to region NOB. It is evident that for prescribed σ and L the less the ratio of the right-hand part to the left (N/L) compared with one (line OB), the greater the reserve of EE compared with the work required for breaking the cube, and conversely the greater this ratio, the more reliable is the possibility of its failure excluded. Therefore, beams emerging from point O on the N vs L diagram in region NOB relate to a condition with an identical "safety factor," and in region BOL they characterize the degree of risk of possible failure. Horizontal lines N_i (assuming that E and λ do not depend on L), relate to different σ_i. With $N = \infty$, $\sigma = 0$, and with $N = 0$ the physically unachievable value $\sigma = \infty$. For a cube with prescribed L_i gradual loading from $\sigma = 0$ to $\sigma = \sigma_i$ corresponds to movement along vertical $L = L_i$ from infinity to $N_i = 2\lambda E/\sigma_i^2$.

By finding line N with $\sigma = \sigma_y$ ($N_0 = 2E\lambda/\sigma_y^2$) we divide each of regions LOB and NOB into two regions (1-4 in Fig. 1). In the upper regions strain is elastic, and in the lower regions it is plastic [according to (1) and (2)]. The value $L = L_0$ at the point of intersection of line OB and $N = N_0$ is determined as the material brittleness threshold. This is the minimum L at which failure is possible in the elastic deformation region.

Thus, the possibility of developing brittleness or ductility for the same material in relation to L is an objective property of the material resulting from the IA to the problem of failure.

Typical values of L_0 calculated for certain materials with $T \sim 300$ K are given in Table 1, whence it follows that in the range of change of L_0 for actual materials extends not less than eight orders of magnitude, and therefore the position of line N_0 in Fig. 1 in contrast to line OB is markedly individual. For a specific material L_0 is not constant since λ, E, and particularly σ_y determining it are functions of temperature T, loading rate $\dot{\sigma}$, or strain rate $\dot{\varepsilon}$. The position of L_0 depends also on the type of stressed state. Thus, the change in tensile forces to compressive leads to brittle failure with stresses greater by a factor of ν^{-1}, and in a plane parallel to the compressive forces a multiple ν^2 (ν is Poisson's ratio) appears in L_0.

TABLE 1

Parameter	Copper	St. 12Kh18N10T (stainless)	st. 3	St. 40Kh (hardened)	Organic glass	Glass
E, GPa	130	200	200	200	3,6	95
$\lambda \cdot 10^{-5}$, J/m^2	0,3	4,7	1,0	1,3	0,12	$92 \cdot 10^{-5}$
σ_y MPa	30,6	350	250	986	120	>3800
L_0, m	8,33	1,53	0,64	0,053	0,006	$<1{,}2 \cdot 10^{-6}$

We move to characteristic regions 1-4 in the N vs L diagram.

Region 1. Nonfailure condition (4) is fulfilled. All states of the region are safe for cube operation. With stable uniform conditions with an increase in L (changeover from state a_0 to a_1) or with an increase in σ (changeover from a_0 to a_2) new states appear on the beams for less strength reserve. Possible values of L_0 are bounded by line OB, and for σ_y, $L = L_0$. With a change to stronger material or with a change in loading conditions (T, $\dot{\sigma}$) leading to an increase in σ_y with unchanged λ and E, the value of L_0 falls. In particular, for pulsed loads when toughness component σ_y arises, and in region 1 a cube of fluidity may appear, although for fluidity under static conditions $\sigma_y = 0$.

For this region the most marked processes changing values of λ, σ_y, and E for the material are slowly flowing processes of structural defect accumulation, i.e., aging, as a result of which the state of the cube may move into another region.

Region 2 (assemblage of elastic deformation and brittle failure states). Necessary condition for failure (3) is fulfilled. The flatter the beam OB for the point characterizing the state of the cube, the greater the excess of the plastic energy safety factor over the work for failure, the lower the measure of material damage necessary in order to fulfill the sufficiency condition, and the more intense will be the process of failure. However, if the sufficiency condition is not fulfilled (there is no critical Griffiths crack, structural defect, anomalous region of overstressing, etc.), failure does not occur. Therefore, if the diagnostics for defects during operation for the cube in question are at a quite high level, and failure of it does not lead to catastrophic consequences, then the condition of region 2 or part of it adjacent to line OB may also be considered as suitable for operation.

Failure of two cubes of different size L_1 and L_2 in the states of region 2 may be accompanied by strong scale effects (SE). If the states are located on beam OB with the same degree of risk (points b_1, b_2), then for them inequality (3) may be written as

$$\sigma^2 L^3/(2E) = A\lambda L^2, \tag{5}$$

where A is ratio of EE safety margin to the work of failure for line OB_i (for OB, A = 1). By substituting values of L_1 and L_2 in (5) and taking their ratio, we obtain

$$\sigma_1/\sigma_2 = \sqrt{L_2/L_1}. \tag{6}$$

Thus, with fulfillment of Eq. (5) and the sufficiency condition, failure of different size cubes will occur with different σ. It is also evident that cubes of the same size but having defects of different size will fail with different stresses relating to lines of the different degree of risk (points b_2, b_3, b_4). This dispersion of brittle strength, which is contrary to that expected, is not described by statistical strength theory [15]. If the conditions for cubes L_1 and L_2 are described by points on different beams, then this may either strengthen the SE (points b_1, b_4), or weaken it (points b_1, b_3) compared with that predicted by Eq. (6). It is noted that if a cube is prepared from traditional plastic material (plasticity is determined by available standard methods) but its condition lies in region 2, then it is not possible to exclude brittle failure. With stable uniform conditions the probability of this event will grow together with L.

Region 3. Here, as in region 2, the necessary condition for failure is fulfilled. Failure in this region is preceded by plastic deformation which leads to the following characteristics: a) λ, E, ν changing weakly in regions 1 and 2 as structural defects increase with an increase in ε affect functions ε and t. The description of cube failure is markedly complicated and it requires knowledge of these functions; b) with an increase in ε there is a sharp increase in internal friction for the material making conditions for transfer of EE difficult; there is a reduction in the neighborhood of the failure zone from which it is possible to remove EE. Therefore if the neighborhood of N_0 and SE and dispersion of brittle strength are still possible, then with departure from N_0, these effects at least with static loading should disappear. Accumulation of macroscopic defects ahead of failure should acquire a more local and aggregate character; c) with relatively small ε the nature of failure is maintained in the form of material breakage. However, with an increase in ε a changeover to shear failure might be expected along selected weakened planes of structural defect localization. In spite of an increase in the failure surface for a cube in shear, from an energy point of view this failure process will apparently be more favorable; d) with large values of ε, particularly with a dynamic effect, the proportion of energy expended directly in failure ($\sim\lambda L^2$) is negligibly small compared with the plastically dissipated and kinetic parts of the energy.

Region 4. As for region 1, for points of this region the nonfailure condition is fulfilled, but in contrast to region 1 the cube material is subject to irreversible strains which in a number of cases limits the possibility of its operation, and as shown above, complicates the description process. In region 4 processes of accumulation and increase in material structural defects with time are typical, i.e., processes of aging and so-called dissipation of failure within the volume of the material, as also for region 3, with high values of ε, but without merging of these zones along some failure surface.

To what extent is this consideration for a hypothetical σ vs ε (1), (2) diagram and an elementary object, i.e., a cube, applicable for real σ vs ε diagrams for materials and more complex objects with not such trivial stressed states? Where is the place in the N vs L diagram for existing strength criteria (concepts, theories)?

The changeover to σ vs ε diagrams for real materials, with the exception of some possible complication in calculations, is not significantly reflected in the N vs L diagram. A changeover to a real object is more complicated. As in the example with a cube, by varying L we assume that all of the dimensions of the object change in a geometrically similar way, and the stressed state is retained (naturally, apart from derivatives with respect to σ and ε along the coordinate and time*). Thus, consideration in the plane N vs L will be limited to considering geometrically similar objects (GSO) loaded identically. For L and σ we take any typical values for a given object (for example, for a spherical vessel as L it is possible to take any of its radii, and for σ it is possible to take its maximum value with selected R). With a complex stressed state for σ it is possible to take a value of relative stress. The permissible arbitrariness in determining L and σ is balanced by introducing coefficients C_i instead of 2 in expressions (3) and (4). Coefficient A, as previously in [5], is the ratio of reserve of EE to the work for failure. It is apparent that for each form of the GSO considered loaded identically the position of line N_{0i}, similar N_0, as also for L_{0i} similar to L_0, will also be its own. For a number of cases a knowledge of C_i is not obligatory. For example, we find L_{0i} for soft steel with spalling failure. The effective value of σ_y is $\sigma_{ef} = K_1K_2K_3\sigma_y$, where $K_1 = (1-\nu)/(1-2\nu)$ considers the change in stressed state; $K_2 = 4.5$ considers the change in σ_y as a result of high-velocity shock loading; $K_3 = 2$ considers unloading wave propagation through a previously compressed material. Then $\sigma_{ef} \sim 16\sigma_y$ and $L_{0i} \simeq 2.5$ mm instead of $L_0 = 640$ mm. Failure of soft steel by spalling should be considered as brittle, and its description is given in [16].

How is it possible to supplement characteristics of regions 1-4 of Fig. 1 with a changeover to the simplest GSO and where is it possible to use the most widespread strength criteria? It is evident that all critical objects or their supporting assemblies whose failure is fought with catastrophic consequences should relate to the conditions of region 1 and not only under normal conditions of prolonged loading, but also with accident situations and extreme loads. Since the conditions of region 1 relate to lightly loaded objects, then a reduction in their specific material content, weight, and an increase in load may be achieved by total or partial use of a directional composite or rolled material free from a SE of an energy nature, or where possible substitution of a single object by a larger number of GSO with a lower value of L [8, 9].

In determining boundaries for the region, and in particular line OB, complications may arise if instead of λ use is made of its analog, i.e., the value of 2γ determined by the FM

*The difference in behavior of materials as a result of nonconformity of values of $\dot{\sigma}$ and $\dot{\varepsilon}$ for GSO is small and it cannot be considered. For steels with changes in $\dot{\varepsilon}$ by a factor of ten the σ_y changes by not more than 5%.

method. According to [17] λ and 2γ, which coincide with low T, diverge with an increase in T. The reason for this behavior is apparently connected with incorrect determination of 2γ. With normal T for soft steels 2γ may exceed λ by a few factors.

In this region use of strength criteria which are traditional for the heading of material strength based on limiting values of σ, ε, $\sigma^2/2E$ is permissible. It is possible to advance as these values $\sigma = \sigma_y$ and $\varepsilon = \sigma_y/E$, and relatively they may be determined for the "safety factor." In fact, these values of safety factor as noted previously depend on the position of points governing the state of GSO. For states placed on a single beam of equal safety factor B_k (point a_1 and a_2) traditional values, for example failure σ, may differ markedly. An object from state a_1 may fail in a brittle way with $\sigma < \sigma_y$, and from state a_2 it may only fail in the region of plastic flow with $\sigma > \sigma_y$. Even greater is the difference in safety factors for different size objects loaded with the same intensity (points a_0 and a_1).

We move to region 2. Creation and use of high-strength materials (with high σ_y) with the aim of easing construction on one hand and an attempt to develop even larger objects on the other, leads to the situation that region 2 of the N vs L diagram is even more important for technology and in fact for this region the problem of brittle failure has appeared in the last 10 years as a central one and led to the vigorous development of FM. Since the required condition for failure in region 2 is fulfilled, it is particularly important to acquire diagnostics for defects with the aim of not permitting fulfillment of the sufficiency condition for changeover of an object into an equilibrium state, i.e., failure. In order to understand the degree of possible risk, information is important about the value of coefficient A for the object in question. Loading rate for the object has a strong effect on the state of this region. Experiments for high-velocity pulsed failure of vessels and spalling separation of a material provide a basis for assuming that with quite intense loading fulfillment of the required failure condition (3) automatically leads to satisfaction of the sufficiency condition [6, 8]. The validity of this assertion follows from the coincidence of the start of failure for selected lines and even planes [14, 18], and high-velocity failure itself may exhibit a number of features. Thus, such specific shock-wave failure as spalling logically falls within the general N vs L diagram [3, 16]. Some deviation from it for specific materials is explained by the unconsidered influence of such effects as phase transition, a different degree of dynamic strengthening for different materials, etc. In total conformity with that stated previously are strong SE with failure and appearance of dispersion for brittle strength in region 2. For states with a large excess of EE ($A \gg 1$) failure will occur by a type of explosion with formation of a large number of fragments [19].

These conditions are energetically favorable for realizing processes of crushing and grinding. In fact, in analyzing the states of region 2 the greatest success of FM is achieved.

In substituting a cube by other GSO all of that said previously in relation to region 3 of the N vs L diagram is entirely retained. In region 2 with small plastic strains (0.5-1.0%) strong SE develop, which was observed with standard static tests in [5, 20]. With dynamic failure of different thickness elliptical vessels made of St. 22K [6] with strains up to 1%, other effects were observed besides development of strong SE. Brittle failure of vessels, which is typical in the elastic region of strains (region 2), becomes ductile with a reduction in vessel size.* There is also a cardinal change in the position of a weak area of a vessel. With failure in the elastic region, crack generation, development, and propagation occur from the place of greatest stress concentration perpendicular to the weld and the thinnest area of the shell. With failure under plastic deformation conditions a crack propagates through a thin area of the shell parallel to a welded joint.

An increase in dissipative losses with an increase in the degree of plastic deformation makes occurrence of wave processes difficult and it severely limits the region from which it is possible to remove EE in forming breakage. The extent of this region ceases to depend on the characteristic size of the object, and an SE of an energy nature disappears. In fact, this is confirmed in standard static tests for tensile failure of different size specimens of "ductile" materials. A more careful study of the failure of specimens made of St. 12Kh-18N10T showed existence of a weak SE (change in $\Delta\sigma \sim 5\%$ with $L_1/L_2 = 6$), apparently of a production nature. Transfer to a high-velocity dynamic effect markedly complicates the occurrence of processes and leads to an increase in EE as a result of an increase in σ_y, and

*Failure of equally thick shells and with strain at 1.5% remained brittle [21].

simultaneously time limits are imposed on the process of removing EE and completion of failure. Analysis of the phenomena from the position of IA made it possible to describe theoretically and to confirm experimentally existence of dynamic peak in ductility for soft steels [4, 11-13]. A study of some deformation equations showed various behavior of materials with dynamic failure and the possibility of describing high-velocity fragmentation with large plastic strains for cylindrical shells and cumulative jets [7, 10].

Returning to existing strength theory, it should be concluded that in the elastoplastic region deformation with not very high values of ε there is currently successful use of FM and modifications of it [22], and in the region of high plastic strains with static loads during creep, etc., there is successful use of traditional strength theories based on using critical values of ε, σ, and combinations of them.

It is substantially more complicated to use FM in order to describe failure of object with high strain rates under conditions of intense loading when the object fails and it is not possible to indicate the weak area beforehand. Shells and also rings loaded suddenly from within by an applied pressure may be related to these objects.

In the last 10 years criteria based on the Taylor approach have been added to the succession of traditional failure criteria based on critical values of ε and σ. According to this approach failure of a shell sets in immediately, as soon as hoop stresses over the whole cross section become tensile. However, these criteria are in contradiction with experiments. In fact, two geometrically similar shells loaded in a similar way (taking account of small correction for the difference in $\dot{\varepsilon}$) should fail with ε = const independent of R, and a pulsed rapidly expanding ring should fail immediately on completion of pulse operation. The experiment is in sharp contradiction to the prognosis. In the Taylor approach there is no place for the dynamic peak of plasticity for which experimental confirmation has been obtained in [11, 12, and others].

Region 4 of the N vs L diagram for more complicated objects than cubes may be described by theories for damage accumulation within the volume of the material of the kinetic strength concept (KSC) type and to a lesser extent to its analog

$$\dot{\varepsilon}\tau = \text{const}. \tag{7}$$

This relationship is far from always being correct. In (7) τ is material endurance with a prescribed value of $\dot{\varepsilon}$. The scattered failure arising (failure within the volume of the material) does not contradict nonfulfillment of the necessary loading condition (3). The process of combining scattered failure within the volume into a global crack may be considered as a consequence of gradual creep over the point describing the initial state of the object (point a_4 of region 4 or even point a_3 of region 1), and in region 3 (points b_6, b_7) as structural defects accumulate.

Close in substance to the KSC is a semiempirical approach to studying the kinetics of increase in material defectiveness after different actions up to achieving a critical value with which failure occurs. This approach [23], as also the KSC, has been used in recent years by a number of authors in order to describe spalling failure. Since one and the same material may fail both in region 2 (point b_4), i.e., with $\sigma \ll \sigma_y$, where material defectiveness is low, and in region 3 (point b_6) with $\sigma \gg \sigma_y$, then it is evident that the degree of defectiveness as a failure criterion may only be used in a narrow range of change in initial conditions.

Thus, the scheme suggested for building a general failure theory on the basis of the energy IA makes it possible:

to combine in a single scheme both brittle failure with static [9] and extremely high dynamic loads [3], and failure in the region of high ductility with high strain rates [4];

to understand the place of a number of criteria (theories, concepts) for strength in this scheme by confirming the thesis that the nature of failure is common;

to formulate conditions for safe construction of objects;

to confirm the possibility of the development of strong SE of an energy nature in explicit form with failure of GSO loaded dynamically [5, 6]. These effects with static loads appear in terms of dispersion of brittle strength, and for ductile matrials as unexpected brittle failure for large objects [9];

to look more critically at actual safety factors for large objects planned without consideration of the possible development of SE of an energy nature;

to conclude that such material properties as brittleness and ductility with stable uniform conditions depend on the characteristic size of an object, and to introduce an understanding of brittleness threshold;

to exclude the possibility of development of a SE of an energy nature if with preparation of GSO a characteristic size is retained for the force element. This condition relates to unidirectional composite materials loaded in the reinforcement direction [14];

to describe and physically comprehend the dependence of failure stress with spalling on time of tensile load operation established by experiment [3, 16]. Analysis of the form of temperature dependence for σ_{spa} makes it possible to conclude the incorrectness of determining K_{Ic} in FM with high T [17];

to understand the physical nature and to describe mathematically the dynamic plasticity peak for materials having a toughness component for strength [4, 7, 11, 12], and to advance in the clarification of the mechanism for crushing of shells and breakup of cumulative jets [10].

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PRESSING OF A COMPACT PLASTIC MATERIAL

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Pressing in a closed mold has been considered by many authors [1-5]. In contrast to others, it is shown in the present work that the compaction process occurs in two stages: in the first deformation it is only in the region adjacent to the piston, and in the second it is in the whole volume of the material. In the first stage around the bottom of the mold there is a rigid (undeformed) zone. The position of the boundary between the rigid and deforming zones depends on the amount of upsetting. The first stage ends when this boundary reaches the bottom of the mold. Presence of a densification front is confirmed by experiment [6].

With relatively low density at the rubbing surfaces the Coulomb friction rule operates. With an increase in density normal pressure and frictional force grow in an unlimited way and at a certain instant reach a maximum value permissible by the flow condition. Then the Coulomb friction rule is not valid and the Prandtl friction rule takes effect. The existence of two friction zones, i.e., Coulomb and Prandtl, at the rubbing surfaces is possible at a certain stage of the process. With a further increase in density the Coulomb zone disappears and the Prandtl rule operates on the whole surface of the mold.

Statement of the Problem. We consider pressing of an axisymmetrical sleeve with an internal rod. We introduce a cylindrical coordinate system (r, θ, z), axis z of which coincides with the axis of symmetry of the pressed article [Fig. 1: 1) mold; 2) piston; 3) rod].

The radial velocity of particles v_r should revert to zero at the surface of the rod and the mold wall, i.e., this value is small. We assume that $v_r = 0$. The corresponding equilibrium equation of obtained by the Hill method [7]. The equation of virtual powers has the form

$$\int_{R_1}^{R_2}\int_0^h \left(\sigma_z \frac{\partial v_z}{\partial z} + \tau_{rz}\frac{\partial v_z}{\partial r}\right) r\,dr\,dz = \int_{R_1}^{R_2} \sigma_z v_z r\,dr\Big|_{z=h} + \int_0^h \tau_{rz} v_z r\,dz\Big|_{r=R_1}^{r=R_2}. \tag{1}$$

Here R_2, R_1 are mold and rod radii; h is current blank height; v_z is projection of velocity on axis z; σ_z, τ_{rz} are normal and tangential stress tensor components.

In order to satisfy boundary conditions at the bottom of the container and base of the piston we assume that v_z does not depend on r, and we perform in the left-hand part of Eq. (1) integration with respect to parts:

$$\int_0^h \left[\int_{R_1}^{R_2} \left(r\frac{\partial \sigma_z}{\partial z} + \frac{\partial (r\tau_{rz})}{\partial r}\right) dr\right] v_z\,dz = 0. \tag{2}$$

Since v_z is a derivative of function z, then from (2) it follows that the expression in square brackets should be equal to zero. The equilibrium equation is written as

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